

Physics-augmented models to simulate commercial adaptive cruise control (ACC) systems

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Introduction

Accurate simulation of mixed traffic, including human-driven vehicles (HDVs) and automated vehicles (AVs), is crucial to the development and assessment of cooperative, connected, and automated mobility (CCAM) technologies. To reproduce driving behaviors of HDVs and AVs, car-following (CF) and adaptive cruise control (ACC) models are developed in the literature of traffic flow. Moreover, these models are augmented to capture essential but overlooked aspects, e.g., vehicle dynamics, perception/actuation delays, and acceleration capabilities. However, a systematic analysis of models augmented with those aspects at different levels of detail is still lacking. This study, therefore, compares the accuracy and robustness of 90 different models, resulting from all possible combinations of five base CF/ACC models and different physics extensions, including vehicle dynamics, acceleration constraints, and perception delay.

Problem description

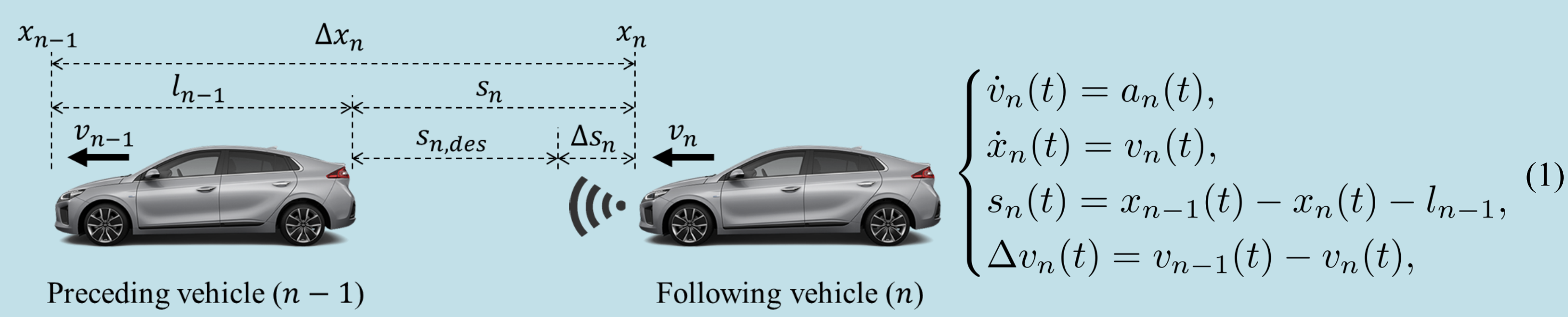


Fig. 1. Kinematics in car-following problems.

The movement of the following vehicle (n) depends not only on the trajectory of the preceding vehicle ($n - 1$), but also on the characteristics of the follower vehicle, driver/ACC, road, etc. In car-following problems, main variables include acceleration (a), speed (v), position (x), and inter-vehicle spacing (s).

Modelling framework

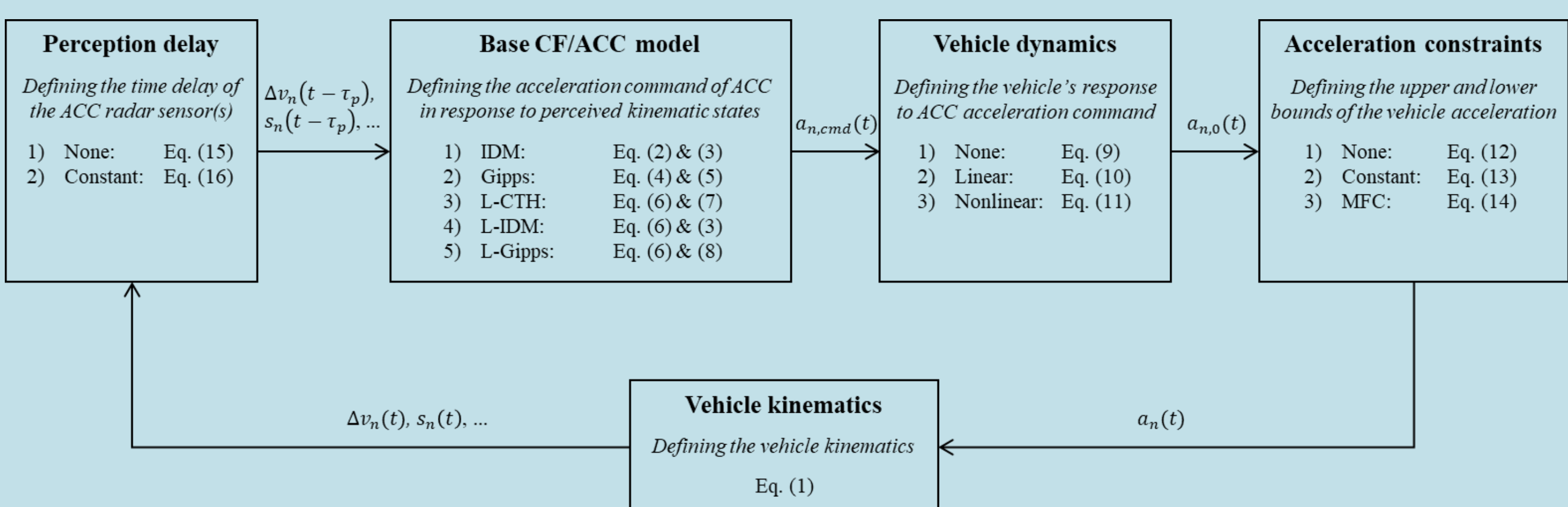


Fig. 2. Unified modelling framework.

I. Base CF/ACC models

1) Intelligent driver model (IDM):

$$a_{n,cmd}(t) = a_{max} \cdot \left(1 - \left(\frac{v_n(t)}{v_0} \right)^\delta - \left(\frac{s_{n,des}(t)}{s_n(t)} \right)^2 \right), \quad (2)$$

$$\text{IDM-desired spacing policy: } s_{n,des}(t) = s_0 + \max \left[0, t_h v_n(t) - \frac{v_n(t) \Delta v_n(t)}{2\sqrt{-a_{max} \cdot a_{min}}} \right], \quad (3)$$

2) Gipps' model:

$$a_{n,cmd}(t) = (v_{n,cmd}(t + t_h) - v_n(t)) / t_h, \quad (4)$$

$$v_{n,cmd}(t + t_h) = \min \left\{ v_n(t) + 2.5 a_{max} t_h \cdot \left(1 - \frac{v_n(t)}{v_0} \right) \left(0.025 + \frac{v_n(t)}{v_0} \right)^{0.5}, \right. \\ \left. a_{min} \cdot \left(\frac{t_h}{2} + \theta \right) + \sqrt{a_{min}^2 \cdot \left(\frac{t_h}{2} + \theta \right)^2 - a_{min} \cdot \left(2(s_n(t) - s_0) - t_h v_n(t) - \frac{v_{n-1}(t)^2}{\hat{a}_{min}} \right)} \right\}, \quad (5)$$

3) Linear ACC models (L-CTH, L-IDM, and L-Gipps):

$$\begin{cases} a_{n,cmd}(t) = \min [k_v \cdot \Delta v_n(t) + k_s \cdot \Delta s_n(t), k_0 \cdot (v_0 - v_n(t))], \\ \Delta s_n(t) = s_n(t) - s_{n,des}(t), \end{cases} \quad (6)$$

$$\text{Constant time headway (CTH) spacing policy: } s_{n,des}(t) = s_0 + t_h \cdot v_n(t), \quad (7)$$

$$\text{Gipps-equilibrium spacing policy: } s_{n,des}(t) = s_0 + (t_h + \theta) v_n(t) - 0.5 v_n(t)^2 \left(\frac{1}{a_{min}} - \frac{1}{\hat{a}_{min}} \right), \quad (8)$$

II. Physics extensions

Vehicle dynamics (VD)

$$1) \text{ None: } a_{n,0}(t) = a_{n,cmd}(t), \quad (9)$$

$$2) \text{ Linear (LVD): } \tau_a \cdot \dot{a}_{n,0}(t) + a_{n,0}(t) = a_{n,cmd}(t), \quad (10)$$

$$3) \text{ Nonlinear (NLVD): } \begin{cases} a_{n,0}(t) = \frac{F_t(t) - F_r(t)}{\phi \cdot m}, \\ F_r(t) = f_0 \cdot \cos \alpha(t) + f_1 \cdot v_n(t) + f_2 \cdot v_n(t)^2 + mg \cdot \sin \alpha(t), \\ F_t(t) = \frac{T_t(t)}{r_w} = m \cdot a_t(t), \\ \tau_a \cdot \dot{a}_t(t) + a_t(t) = a_{n,cmd}(t), \end{cases} \quad (11)$$

Acceleration constraints (AC)

$$1) \text{ None: } a_n(t) = a_{n,0}(t), \quad (12)$$

$$2) \text{ Constant: } a_n(t) = \max \{ a_{lb}, \min [a_{n,0}(t), a_{ub}] \}, \quad (13)$$

Microsimulation free-flow acceleration (MFC) boundary model:

$$a_n(t) = \max \left\{ a_{dp}(v_n(t)), \min [a_{n,0}(t), a_{ap}(v_n(t))] \right\}, \quad (14)$$

Perception delay (PD)

$$1) \text{ None: } \tau_p = 0, \quad (15)$$

$$2) \text{ Constant: } \tau_p > 0, \quad (16)$$

III. Summary of 90 models

Model groups	Model ID	ACC controller	Spacing policy	Perception delay (PD)	Vehicle dynamics (VD)	Acceleration constraints (AC)	Calibration parameters		
L-Gipps-based models	74	Linear contr., Eq. (6)	Gipps-equilibrium, Eq. (8)	None, Eq. (15)	None, Eq. (9)	None, Eq. (12)	$k_v, k_s, k_0, \tau_a, \tau_p, s_0, t_h, \theta, \phi, \beta, \delta_{max}, \delta_{min}$		
	75	Model ID	ACC controller	Spacing policy	Perception delay (PD)	Vehicle dynamics (VD)	Acceleration constraints (AC)	Calibration parameters	
	76	55	Linear contr., Eq. (6)	IDM-desired, Eq. (3)	None, Eq. (15)	None, Eq. (9)	None, Eq. (12)	$k_v, k_s, k_0, \tau_a, \tau_p, s_0, t_h, \theta, \phi, \beta, \delta_{max}, \delta_{min}$	
	77	56	Model ID	ACC controller	Spacing policy	Perception delay (PD)	Vehicle dynamics (VD)	Acceleration constraints (AC)	Calibration parameters
	78	57	Linear contr., Eq. (6)	CTH, Eq. (7)	None, Eq. (15)	None, Eq. (9)	None, Eq. (12)	$k_v, k_s, k_0, \tau_a, \tau_p, s_0, t_h, \theta, \phi, \beta, \delta_{max}, \delta_{min}$	
	79	58	Model ID	ACC controller	Spacing policy	Perception delay (PD)	Vehicle dynamics (VD)	Acceleration constraints (AC)	Calibration parameters
	80	59	Linear contr., Eq. (6)	Gipps, Eq. (4)+(5)	Inherent	None, Eq. (15)	None, Eq. (9)	None, Eq. (12)	$k_v, k_s, k_0, \tau_a, \tau_p, s_0, t_h, \theta, \phi, \beta, \delta_{max}, \delta_{min}$
	81	60	Model ID	ACC controller	Spacing policy	Perception delay (PD)	Vehicle dynamics (VD)	Acceleration constraints (AC)	Calibration parameters
	82	61	Linear contr., Eq. (6)	IDM-desired, Eq. (3)	None, Eq. (15)	None, Eq. (9)	None, Eq. (12)	$k_v, k_s, k_0, \tau_a, \tau_p, s_0, t_h, \theta, \phi, \beta, \delta_{max}, \delta_{min}$	
	83	62	Model ID	ACC controller	Spacing policy	Perception delay (PD)	Vehicle dynamics (VD)	Acceleration constraints (AC)	Calibration parameters
L-IDM-based models	40	19	Linear contr., Eq. (6)	Gipps, Eq. (4)+(5)	Inherent	None, Eq. (15)	None, Eq. (9)	None, Eq. (12)	$k_v, k_s, k_0, \tau_a, \tau_p, s_0, t_h, \theta, \phi, \beta, \delta_{max}, \delta_{min}$
	41	20	Model ID	ACC controller	Spacing policy	Perception delay (PD)	Vehicle dynamics (VD)	Acceleration constraints (AC)	Calibration parameters
	42	21	Linear contr., Eq. (6)	IDM-desired, Eq. (3)	None, Eq. (15)	None, Eq. (9)	None, Eq. (12)	$k_v, k_s, k_0, \tau_a, \tau_p, s_0, t_h, \theta, \phi, \beta, \delta_{max}, \delta_{min}$	
	43	22	Model ID	ACC controller	Spacing policy	Perception delay (PD)	Vehicle dynamics (VD)	Acceleration constraints (AC)	Calibration parameters
	44	23	IDM, Eq. (2)	IDM-desired, Eq. (3)	None, Eq. (15)	None, Eq. (9)	None, Eq. (12)	$\delta, \tau_p, s_0, t_h, \theta, \phi, \beta, \delta_{max}, \delta_{min}$	
	45	24	IDM, Eq. (2)	IDM-desired, Eq. (3)	None, Eq. (15)	None, Eq. (9)	MFC, Eq. (14)	$\delta, \tau_p, s_0, t_h, \theta, \phi, \beta, \delta_{max}, \delta_{min}$	
	46	25	IDM, Eq. (2)	IDM-desired, Eq. (3)	None, Eq. (15)	Linear, Eq. (10)	None, Eq. (12)	$\delta, \tau_p, s_0, t_h, \theta, \phi, \beta, \delta_{max}, \delta_{min}, \tau_a$	
	47	26	IDM, Eq. (2)	IDM-desired, Eq. (3)	None, Eq. (15)	Linear, Eq. (10)	Constant, Eq. (13)	$\delta, \tau_p, s_0, t_h, \theta, \phi, \beta, \delta_{max}, \delta_{min}, \tau_a$	
	48	27	IDM, Eq. (2)	IDM-desired, Eq. (3)	None, Eq. (15)	Linear, Eq. (10)	MFC, Eq. (14)	$\delta, \tau_p, s_0, t_h, \theta, \phi, \beta, \delta_{max}, \delta_{min}, \tau_a$	
	49	28	IDM, Eq. (2)	IDM-desired, Eq. (3)	None, Eq. (15)	Nonlinear, Eq. (11)	None, Eq. (12)	$\delta, \tau_p, s_0, t_h, \theta, \phi, \beta, \delta_{max}, \delta_{min}, \tau_a$	
L-CTH-based models	84	42	Linear contr., Eq. (6)	Gipps, Eq. (4)+(5)	Inherent	None, Eq. (15)	None, Eq. (9)	None, Eq. (12)	$k_v, k_s, k_0, \tau_a, \tau_p, s_0, t_h, \theta, \phi, \beta, \delta_{max}, \delta_{min}$
	85	43	Model ID	ACC controller	Spacing policy	Perception delay (PD)	Vehicle dynamics (VD)	Acceleration constraints (AC)	Calibration parameters
	86	44	IDM, Eq. (2)	IDM-desired, Eq. (3)	None, Eq. (15)	None, Eq. (9)	None, Eq. (12)	$\delta, \tau_p, s_0, t_h, \theta, \phi, \beta, \delta_{max}, \delta_{min}$	
	87	45	IDM, Eq. (2)	IDM-desired, Eq. (3)	None, Eq. (15)	None, Eq. (9)	Constant, Eq. (13)	$\delta, \tau_p, s_0, t_h, \theta, \phi, \beta, \delta_{max}, \delta_{min}$	
	88	46	IDM, Eq. (2)	IDM-desired, Eq. (3)	None, Eq. (15)	Linear, Eq. (10)	None, Eq. (12)	$\delta, \tau_p, s_0, t_h, \theta, \phi, \beta, \delta_{max}, \delta_{min}, \tau_a$	
	89	47	IDM, Eq. (2)	IDM-desired, Eq. (3)	None, Eq. (15)	Linear, Eq. (10)	Constant, Eq. (13)	$\delta, \tau_p, s_0, t_h, \theta, \phi, \beta, \delta_{max}, \delta_{min}, \tau_a$	
	90	48	IDM, Eq. (2)	IDM-desired, Eq. (3)	None, Eq. (15)	Linear, Eq. (10)	MFC, Eq. (14)	$\delta, \tau_p, s_0, t_h, \theta, \phi, \beta, \delta_{max}, \delta_{min}, \tau_a$	
	91	49	IDM, Eq. (2)	IDM-desired, Eq. (3)	None, Eq. (15)	Nonlinear, Eq. (11)	None, Eq. (12)	$\delta, \tau_p, s_0, t_h, \theta, \phi, \beta, \delta_{max}, \delta_{min}, \tau_a$	
	92	50	IDM, Eq. (2)	IDM-desired, Eq. (3)	None, Eq. (15)	Nonlinear, Eq. (11)	Constant, Eq. (13)	$\delta, \tau_p, s_0, t_h, \theta, \phi, \beta, \delta_{max}, \delta_{min}, \tau_a$	
	93	51	IDM, Eq. (2)	IDM-desired, Eq. (3)	None, Eq. (15)	Nonlinear, Eq. (11)	MFC, Eq. (14)	$\delta, \tau_p, s_0, t_h, \theta, \phi, \beta, \delta_{max}, \delta_{min}, \tau_a$	

Fig. 3. Overview of Formulas and calibration parameters of 90 models.

Platoon trajectories in JRC OpenACC database

The test platoon was composed of five high-end vehicles equipped with commercial ACC. All the data used are part of the JRC OpenACC database openly available online.

Table 1. Composition of platoons.

Vehicle position	P1	P2	P3	P4	P5	P6	P7
Leader	Audi A8	Audi A8	Audi A8	Audi A8	Audi A8	Audi A8	Audi A8
Follower 1	Audi A6	Audi A6	Tesla	Tesla	Tesla	Mercedes	Mercedes
Follower 2	BMW	BMW	BMW	BMW	BMW	BMW	BMW
Follower 3	Mercedes	Mercedes	Audi A6	Audi A6	Audi A6	Tesla	Tesla
Follower 4	Tesla	Tesla	Mercedes	Mercedes	Mercedes	Audi A6	Audi A6

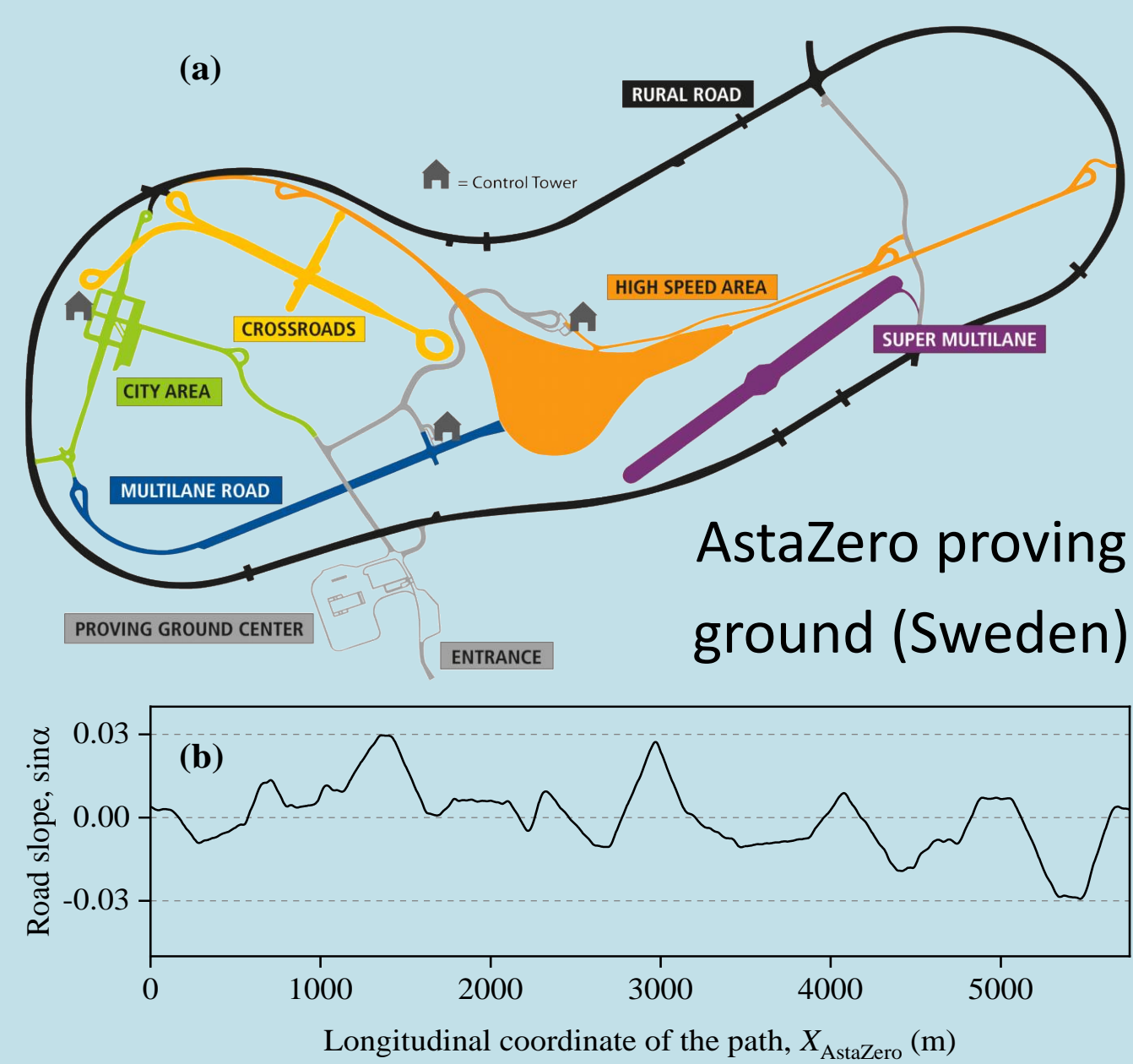


Fig. 4. Layout (a) and slope (b) of the test track.

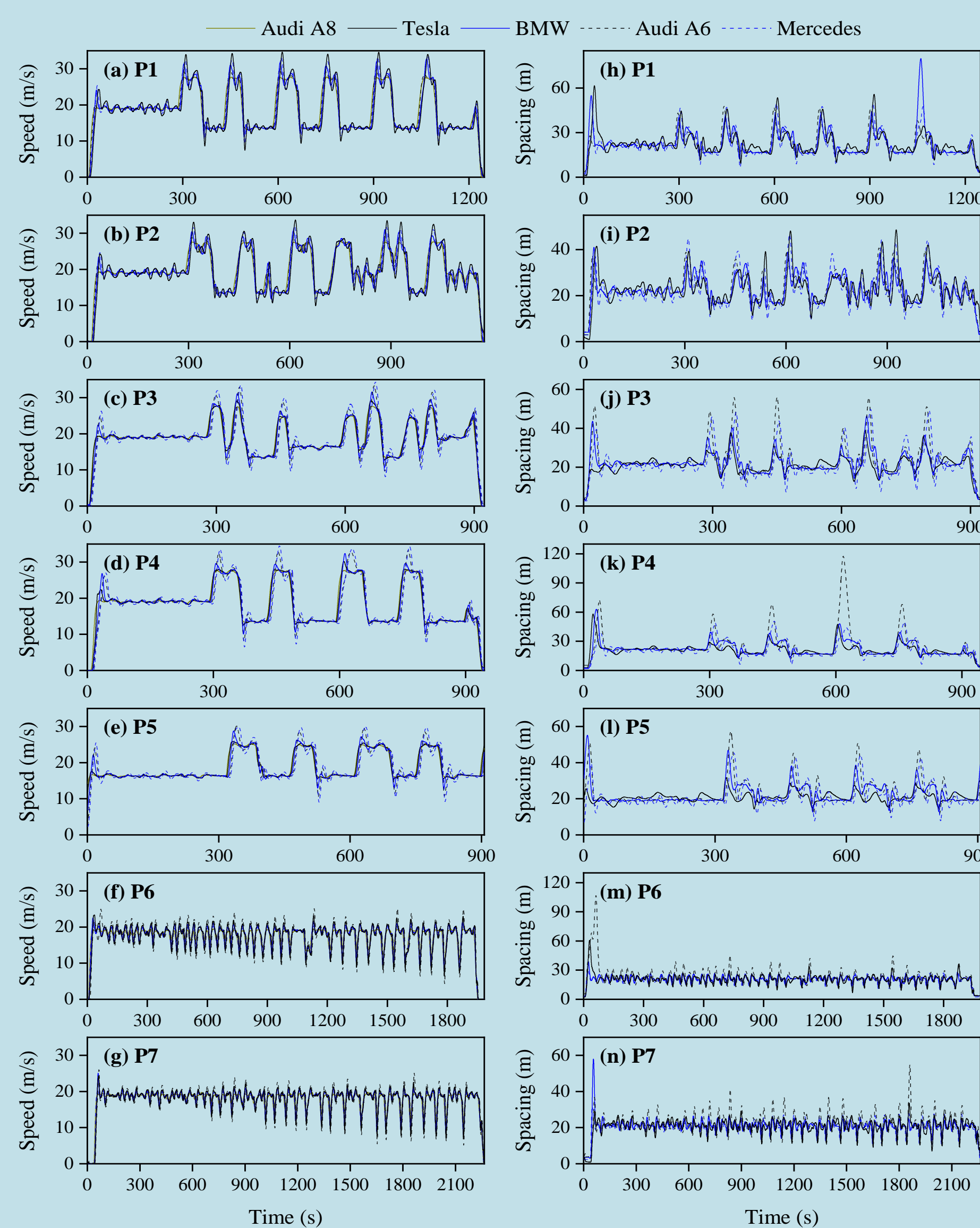


Fig. 5. Speed (left) and inter-vehicle spacing (right).

Results

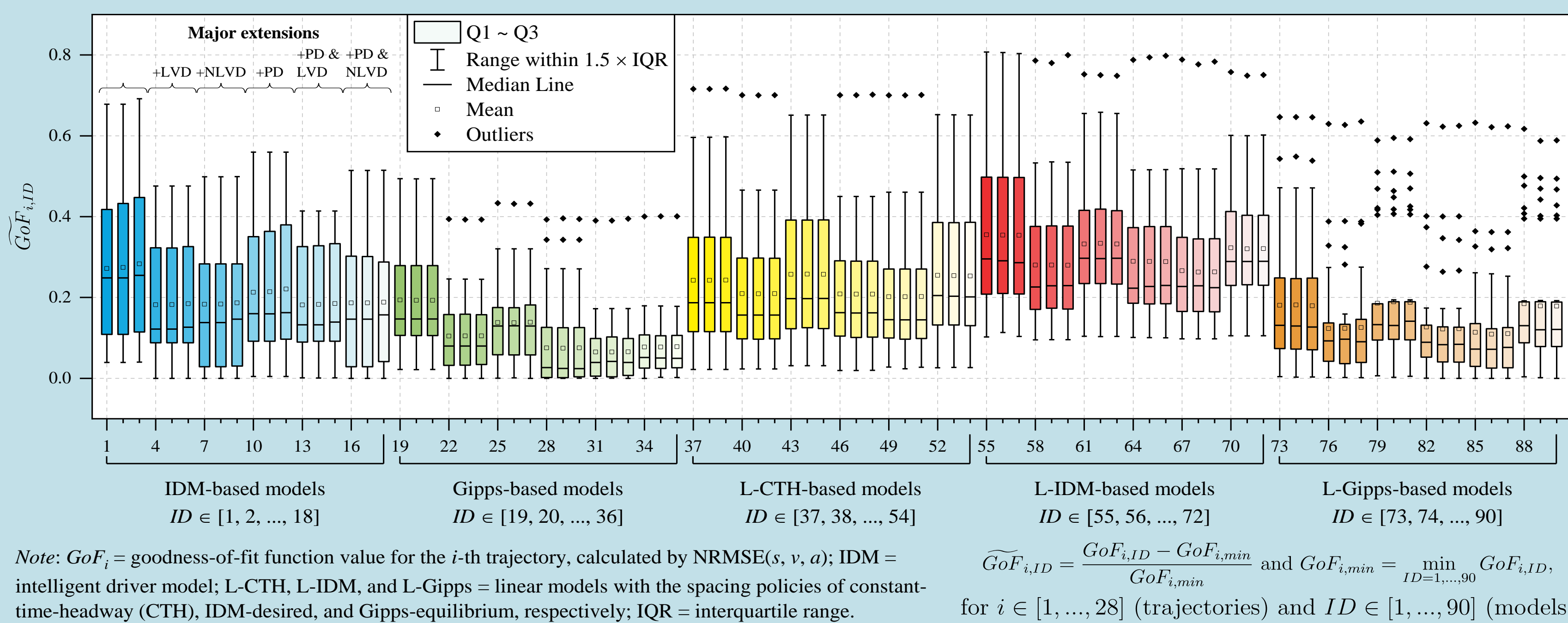


Fig. 6. Variability of normalized calibration errors ($\widetilde{GoF}_{i,ID}$) within (individual box plot) and among models

- Variance-based sensitivity analysis: $S_{trajectories} = 0.819 > S_{models} = 0.062$.
- Gipps-based models demonstrate the best performance in calibration experiments.
- Perception delay (PD) and linear vehicle dynamics (LVD) can improve model accuracy.
- Nonlinear vehicle dynamics (NLVD) can provide accuracy benefit to IDM- and Gipps-based models.

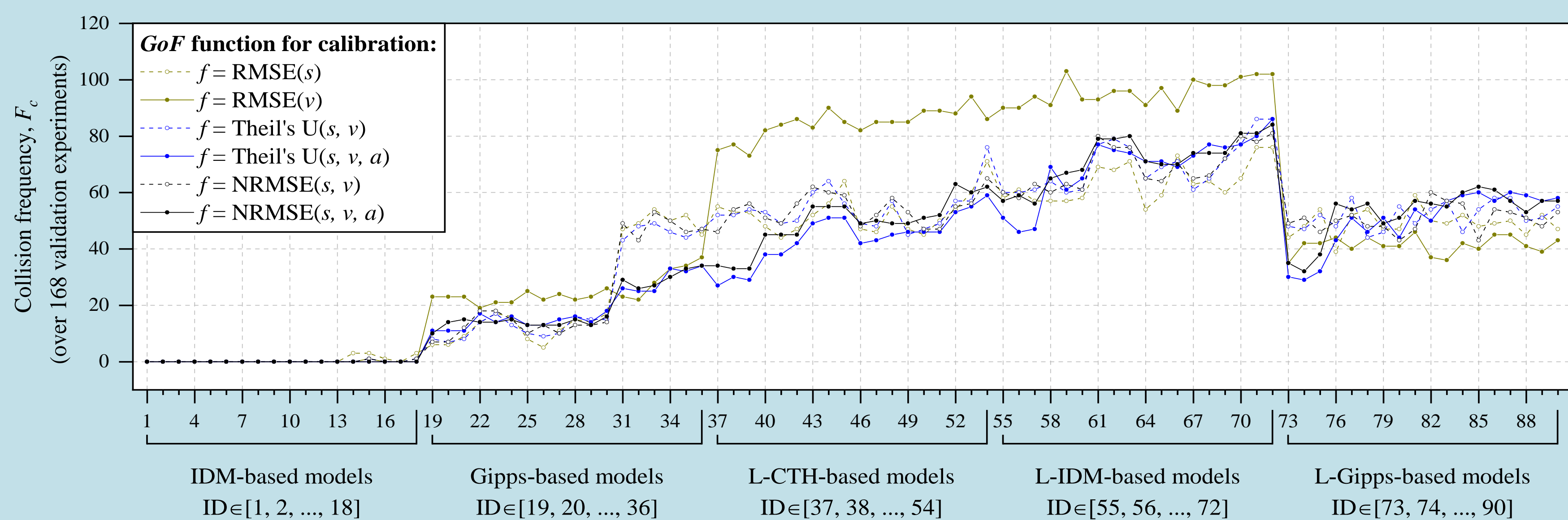


Fig. 7. Collision frequency (for each model variant) over 168 validation experiments.

- $NRMSE(s, v, a)$ and Theil's $U(s, v, a)$ lead to the least number of collisions in validation tests.
- IDM-based models outperform others regarding transferability of parameters and model robustness.

Design of experiments

I. Optimization problem

$$\min_{\beta} f(Y^{obs}, Y^{sim}(\beta)),$$

$$Y^{sim}(\beta) = F(\beta),$$

$$LB_{\beta} \leq \beta \leq UB_{\beta}, G(\beta) \leq 0,$$
(17)

β is the set of calibration parameters; F is the model; LB, UB , and G are constraints.

II. Evaluation metric

Goodness-of-fit (GoF) function: $f = NRMSE(Y)$,

Measure of performance (MoP): $Y \in [s, v, a]$,

$$\begin{cases} NRMSE(s, v, a) = w_0 NRMSE(s) + w_1 NRMSE(v) + w_2 NRMSE(a), \\ NRMSE(Y) = RMSE(Y) / \sqrt{\frac{1}{N} \sum_{i=1}^N (Y_i^{obs})^2}, \\ RMSE(Y) = \sqrt{\frac{1}{N} \sum_{i=1}^N (Y_i^{sim} - Y_i^{obs})^2}, \end{cases}$$
(18)

Table 2. Bounds of calibration parameters.

Parameter [unit]	Lower bound (LB)	Upper bound (UB)
δ	0.1	10
v_0 [m/s]	30	35
s_0 [m]	1	5
t_h [s]	0.1	3
a_{max} [m/s ²]	0.5	5
a_{min} [m/s ²]	-5	-0.5
\hat{a}_{min} [m/s ²]	-5	-0.5
θ [s]	0	3
τ_a [s]	0.3	0.8
τ_p [s]	0.1	0.8
k_s [s ⁻²]	0.01	5
k_v [s ⁻¹]	0.01	5
k_0 [s ⁻¹]	0.01	5

Conclusions

I. Contributions

- A comparison framework was proposed to assess CF/ACC models augmented at different levels of detail with vehicle dynamics, acceleration constraints, and perception delay. These physics extensions were essential aspects of human/automated driving behavior.
- Two new spacing policies derived from microscopic traffic flow theory (TFT) were coupled with linear ACC controllers.
- Calibration experiments help to assess the contribution of each physics extension to the accuracy of the base model.

II. Conclusions

- Across all 90 model variants, Gipps-based models demonstrate the best performance in reproducing ACC driving trajectories.
- In terms of transferability of parameters and model robustness to vehicle collisions in validation experiments, IDM-based models outperform all other models, followed by Gipps-based ones.
- The objective (or GoF) function of $NRMSE(s, v, a)$ can sensibly improve model robustness and transferability of calibrated parameters.
- Perception delay (PD) and linear vehicle dynamics (LVD) can largely improve model accuracy.
- Nonlinear vehicle dynamics (NLVD) can provide accuracy benefit to IDM- and Gipps-based models.